education

Department of Education
FREE STATE PROVINCE

PREPARATORY EXAMINATION

GRADE 12

MATHEMATICS P2

SEPTEMBER 2020

MARKS: 150

TIME: 3 HOURS

This question paper consists of 13 pages, an information sheet and an answer book of 20 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, etc. which you have used in determining your answers.
4. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. Answers only will NOT necessarily be awarded full marks.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The table below gives the average exchange rate and the average monthly oil price for the year 2010.

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
Exchange rate in R/S	7.5	7.7	7.2	7.4	7.7	7.7	7.6	7.3	7.1	7.0	6.9	6.8
Oil price in $	69.9	68.0	72.9	70.3	66.3	67.1	67.9	68.3	71.3	73.6	76.0	81.0

1.1 Draw a scatterplot to represent the exchange rate (in R/S) versus the oil price (in $). (3)

1.2 Determine the equation of the least square regression line. (3)

1.3 Calculate the value of the correlation coefficient. (1)

1.4 Comment on the strength of the relationship between the exchange rate (in R/S) and the oil price (in $). (2)

1.5 Determine the mean oil price. (1)

1.6 Determine the standard deviation of the oil price. (1)

1.7 Generally there is a concern from the public when the oil price is higher than two standard deviations from the mean.

In which months would the public have been concerned? (2)
[13]

QUESTION 2

The average percentage of 150 learners for all their subjects is summarised in the cumulative frequency below.

PERCENTAGE INTERVAL	CUMULATIVE FREQUENCY
$0 < x \leq 10$	5
$10 < x \leq 20$	21
$20 < x \leq 30$	50
$30 < x \leq 40$	70
$40 < x \leq 50$	88
$50 < x \leq 60$	110
$60 < x \leq 70$	135
$70 < x \leq 80$	142
$80 < x \leq 90$	147
$90 < x \leq 100$	150

2.1 Draw the ogive (cumulative frequency graph) to represent the above data on the grid provided in the ANSWER BOOK. (4)

2.2 Use the ogive to approximate the following:

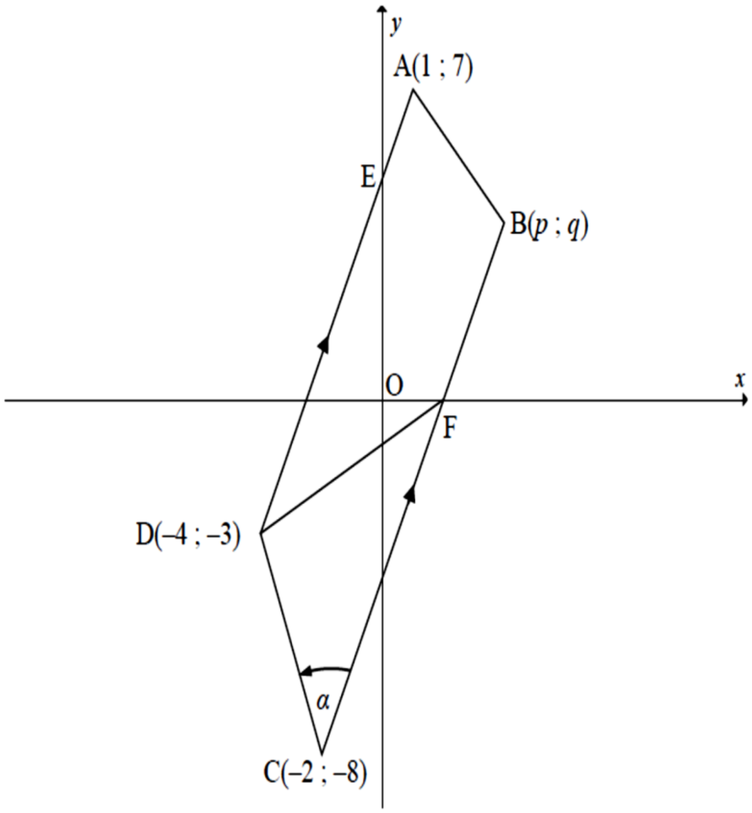
2.2.1 The number of learners who scored less than 85% (2)

2.2.2 The median (1)

[7]

QUESTION 3

In the diagram below shows, quadrilateral ABCD with AD // BC. The coordinates of the vertices are $A(1;7); B(p;q); C(-2;-8)$ and $D(-4;-3)$. BC intersects the x-axis at F. $D\hat{C}B = \alpha$.

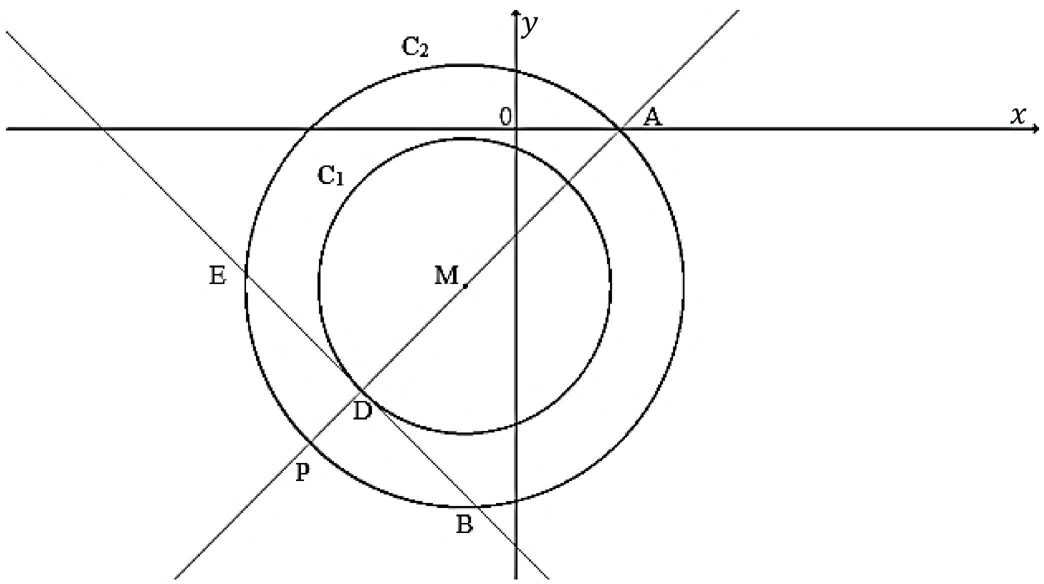


3.1	Calculate the gradient of AD.	(2)
3.2	Determine the equation of BC in the form $y = mx + c$.	(3)
3.3	Determine the coordinates of point F.	(2)
3.4	AB′CD is a parallelogram with B' on BC. Determine the coordinates of B', using a transformation $(x; y) \rightarrow (x + a; y + b)$ that sends A to B'.	(2)
3.5	Show that $\alpha = 48{,}37°$.	(4)
3.6	Calculate the area of ΔDCF.	(6)
		[19]

QUESTION 4

Circle C_1 and C_2 in the figure below have the same centre M. P and A are points on C_2.

PM intersects C_1 at D. The tangent BD to C_1 intersects C_2 at B and E. The equation of circle C_1 is given by $x^2 + 2x + y^2 + 6y + 2 = 0$ and the equation of line PM is $y = x - 2$.



4.1 Calculate the coordinates of centre M. (3)

4.2 Determine the radius of circle C_1. (1)

4.3 Determine the coordinates of D_1 the point where line PM and circle C_1 intersects. (5)

4.4 Give a reason why $M\widehat{D}B = 90°$. (1)

4.5 If is given that $DB = 4\sqrt{2}$, determine MB, the radius of circle C_2. (4)

4.6 Write down the equation of C_2 in the form $(x - a)^2 + (y - b)^2 = r^2$. (1)

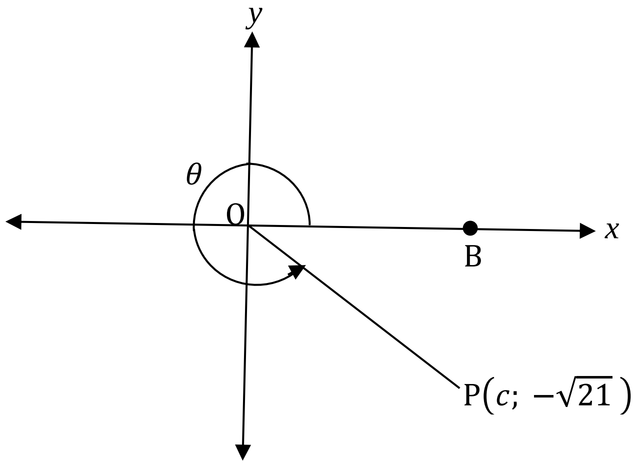
4.7 Is the point $F\left(2\sqrt{5}\,; 0\right)$ inside circle C_2? Support your answer with calculations. (4)

4.8 Determine the gradient of the tangent to circle C_2 at point P. (2)

[21]

QUESTION 5

5.1 In the diagram, P is the point $(c; \sqrt{21})$ such that OP = 5 units.
$B\hat{O}P = \theta$ as indicated.



5.1.1 Calculate the numerical value of c. (2)

5.1.2 Determine **(without the use of a calculator)**, the numerical value of the following:

a) $\cos\ \theta$ (1)

b) $\tan\theta + sin^2\theta$ (2)

c) $\sin 2\theta$ (2)

5.2 Simplify **(without the use of a calculator)**:

$$\frac{\sin(x-180°).\tan x.\cos 690°}{\cos^2(x-90°)}$$ (5)

[12]

QUESTION 6

6.1 Prove the identity: $\frac{2\sin^2 x}{2\tan x - \sin 2x} = \frac{\cos x}{\sin x}$ (5)

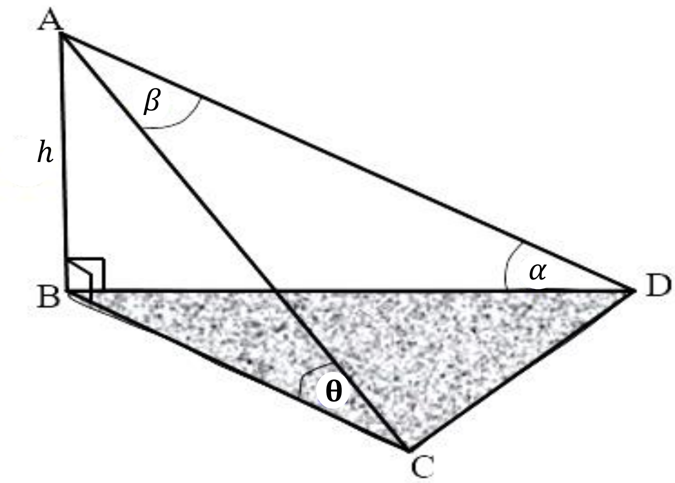
6.2 Show that: $Sin^2 20° + Sin^2 40° + Sin^2 80° = \frac{3}{2}$

(Hint: $40° = 60° - 20°$ and $80° = 60° + 20°$) (7)

[12]

QUESTION 7

In the figure below Thabo is standing at a point A on top of building AB that is h (m) high. He observes two cars, C and D that are in the same horizontal plane as B. The angle of elevation from C to A is θ and the angle of elevation from D to A is α. $C\hat{A}D=\beta$.



7.1 Calculate the length of AC in terms of h and θ. (2)

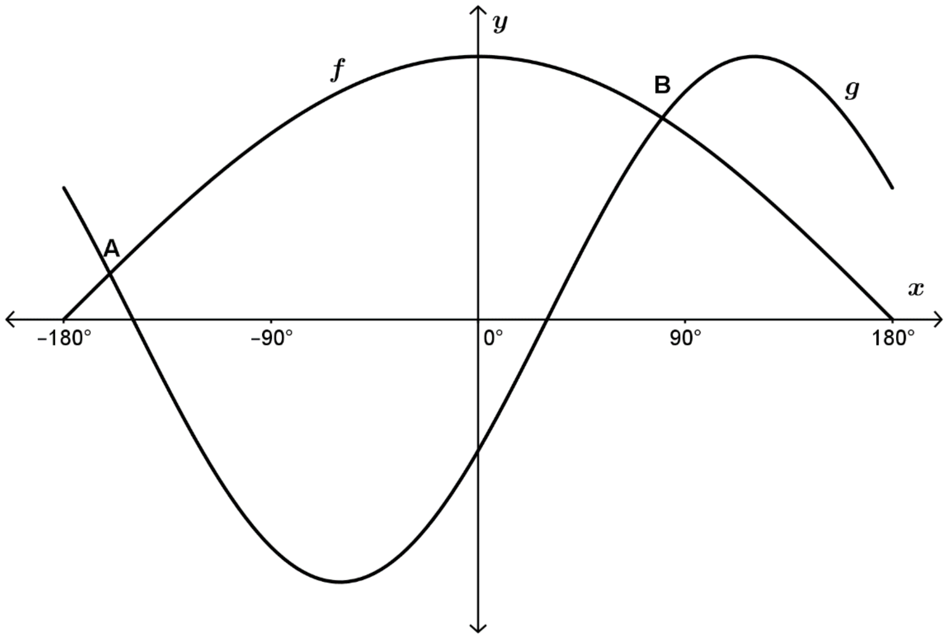
7.2 Calculate the length of AD in terms of h and α. (2)

7.3 Determine the distance between the two cars, which is the length of CD in terms of α, θ, β and h. (3)

[7]

QUESTION 8

In the diagram are the graphs of the functions $f(x) = \cos\frac{x}{2}$ and $g(x) = \sin(x - 30°)$ for $x \, \epsilon[-180°; 180°]$. The curves intersect at A and B.



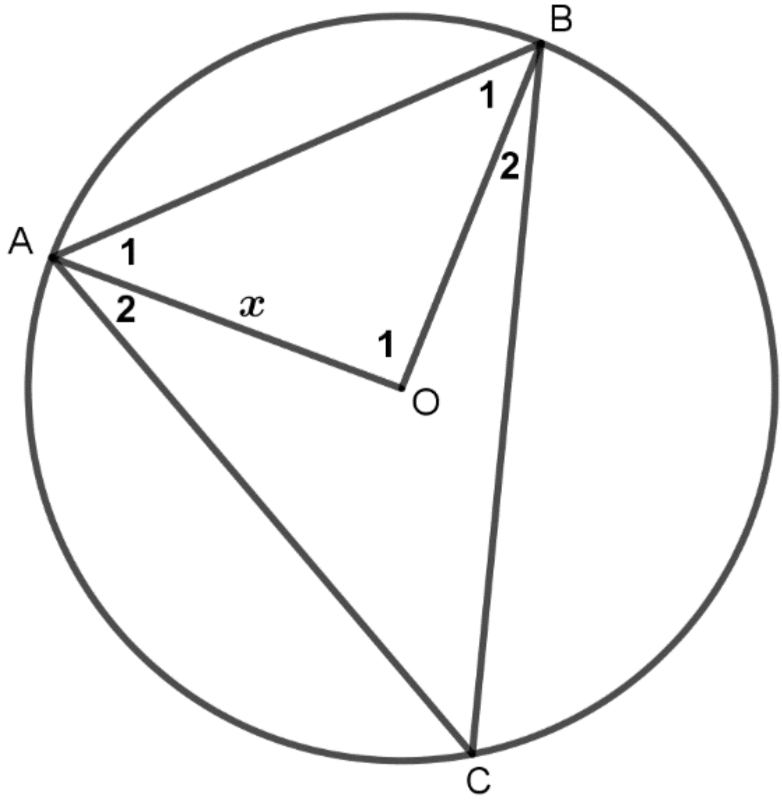
8.1 Calculate the *x*-coordinates of the points A and B. (6)

8.2 Determine the values of $x \, \epsilon[-90°; 180°]$, for which $g(x).f(x) < 0$. (3)

[9]

QUESTION 9

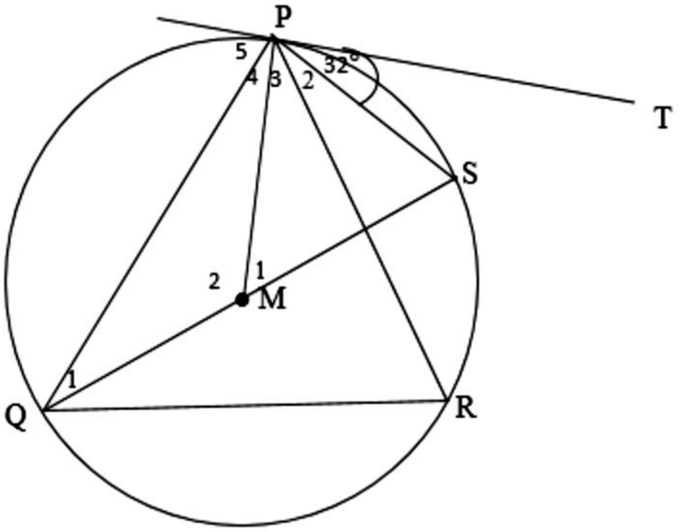
A, B and C are three points on the circle with centre O such that AB = BC = $\frac{3}{2}$ AO.
AO $= x$.



9.1 Calculate the size of $\hat{O}_1$ rounded off to the nearest degree. (5)

9.2 If $\hat{O}_1$ = 97° and $x = 10$ cm, calculate the length of AC correct to two decimal places. (4)

9.3 In the diagram, TP is a tangent to the circle with centre M at P. QS is a diameter of the circle and R is on the circumference of the circle. $T\hat{P}S = 32°$.



Calculate the following, giving reasons:

9.3.1 $\hat{Q}_1$ (2)

9.3.2 $\hat{P}_4$ (2)

9.3.3 $\hat{M}_1$ (2)

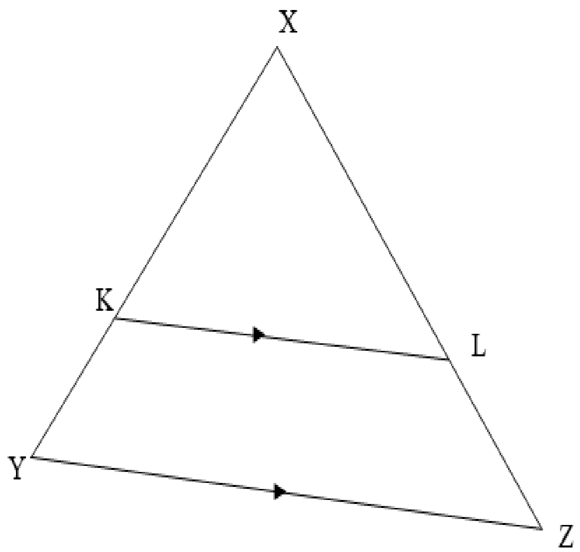
9.3.4 $\hat{R}$ (4)

[19]

QUESTION 10

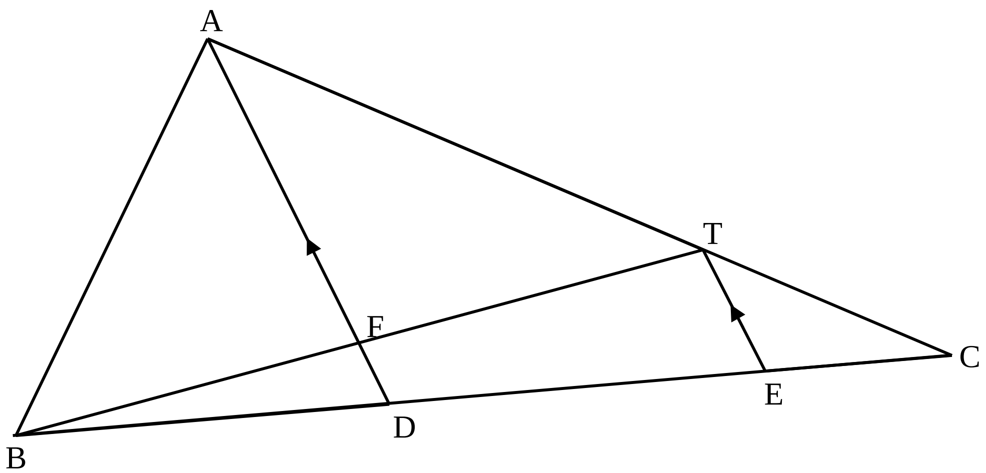
10.1 Complete the following statement:
If two triangles are equiangular, then the corresponding sides are … (1)

10.2 Use the diagram to prove the theorem which states that a line drawn parallel to one side of a triangle divides the other two sides proportionally, that is prove that $\frac{XK}{KY} = \frac{XL}{LZ}$.



(6)

10.3 In the figure, D is a point on side BC of ΔABC such that BD = 6 cm and DC = 9 cm. T and E are points on AC and DC respectively such that TE||AD and AT : TC = 2 : 1



10.3.1 Show that D is the midpoint of BE. (3)

10.3.2 If FD = 2 cm, calculate the length of TE. (3)

10.3.3 Calculate the numerical value of:

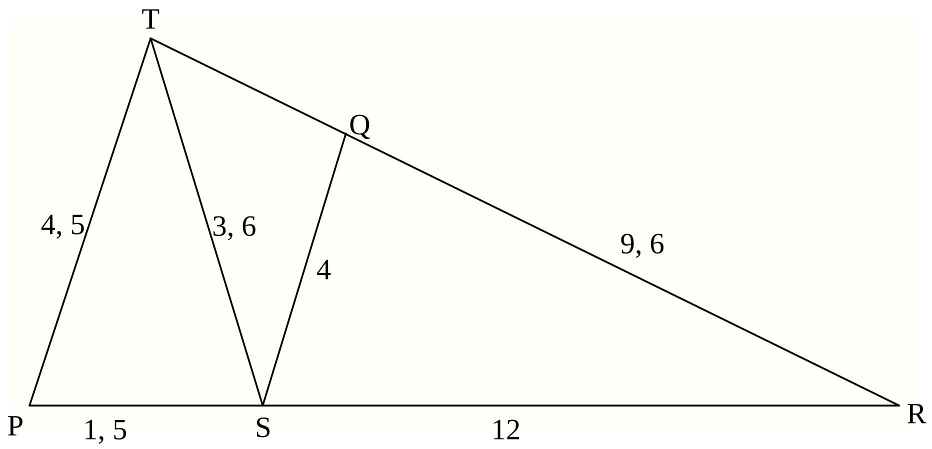
(a) $\frac{\text{Area of } \Delta ADC}{\text{Area of } \Delta ABD}$ (3)

(b) $\frac{\text{Area of } \Delta TEC}{\text{Area of } \Delta ABC}$ (3)

[19]

QUESTION 11

In the diagram, TPR is a triangle with TP = 4,5 units. Points Q and S are on TR and PR respectively such that QR = 9,6 units, QS = 4 units, TS = 3,6 units, PS = 1,5 units and SR = 12 units.



11.1 Prove that PT is a tangent to the circle which passes through the points T,S and R. (7)

11.2 Calculate the length of TQ. (5)

[12]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + in)$ $A = P(1 - in)$ $A = P(1 - i)^n$ $A = P(1 + i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1 - (1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x - a)^2 + (y - b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$